বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2021

(Under CBCS Pattern)

Semester - III

Subject : MATHEMATICS

Paper : C 7 - T & P

Full Marks : 60 (Theory - 40 + Practical - 20)

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

[NUMERICAL METHODS]

(Theory)

Group - A

1. Answer any ***three*** of the following questions : 12×3=36

(a) (i) Factorize the matrix $\begin{bmatrix} 2 & -2 & 1 \\ 5 & 1 & -3 \\ 3 & 4 & 1 \end{bmatrix}$ into the form LU, where L and U are lower and upper triangular matrices and hence solve the system of equations

$$2x - 2y + z = 2$$
$$5x + y - 3z = 0$$
$$3x + 4y + z = 9$$

(ii) Fit a parabola to the following data by taking 'x' as independent variable.

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

8+4

(b) (i) Given $\frac{dy}{dx} = y^2 - x^2$, where $y(0) = 2$. Find $y(0.1)$ and $y(0.2)$ as a solution of this equation by fourth order Runge-Kutta method.

(ii) Solve the following system of equations by Gauss-Seidal method correct up to four significant figures :

$$\begin{aligned} 3x + y + z &= 3 \\ 2x + y + 5z &= 5 \\ x + 4y + z &= 2 \end{aligned}$$

6+6

(c) (i) Consider the equation $5x^3 - 20x + 3 = 0$. Find the root, using iteration method, lying on the interval [0, 1] correct up to 5 decimal places. Find the order of convergence of the iteration method. 6

(ii) Find the value of $\int_0^1 \frac{dx}{1+x^2}$ taking 5 sub-intervals, by trapezoidal rule, correct to five significant figures. Also find the error by comparing with the exact value. 6

(d) (i) Find the method of iteration for numerical integration. 4

(ii) If $x = \alpha$ be a root of the equation $f(x) = 0$ which is rewritten as $x = \phi(x)$. If $\phi(x)$ is continuous and $|\phi'(x)| \le 1$ where $0 < l < 1$, in an interval l containing α, then prove that the sequence (x_n) of iterations determined from $x_{n+1} = \phi(x_n), (n = 0,1,2.....)$ converges to the root α. 4

(iii) Let $y = 5x^7 - 4x$. Find the percentage error in y at $x = 1$, if the error in x is $\Delta x = 0.04$. 4

(e) Find the basic principle for Newton-Raphson method with its geometrical meaning. Find advantages and disadvantages of Newton-Raphson method. How can you use this method for an assigned root of a positive real number.

4+2+2+4

(f) Establish the Gauss Legendre Quadrature formula for numerical integration $\int_a^b f(x)\,dx$ and then establish composite Simpson's $\frac{1}{3}$ rd rule from it. Evaluate $\int_0^1 x^3 dx$, by Simpson's $\frac{1}{3}$ rd rule with $n = 5$. 4+4+4

Group - B

2. Answer any ***two*** of the following questions : 2×2=4

(a) Find $f(x)$, when its first difference is $x^3 + 4x^2 + 2x + 7$.

(b) Define Round off error and Truncation error.

(c) Show that the maximum error in linear interpolation is given by $\frac{h^2 M_2}{8}$ where $M_2 = \max_{0 \le x \le 1} |f'(x)|$.

(d) Compare between Newton-Cote's quadrature and Gaussian quadrature.

(Practical)

[NUMERICAL METHODS LAB]

Group - A

1. Answer any ***one*** of the following questions : 15×1=15

(a) Write a program to find a root of the equation $x^3 - 3x + 1 = 0$ by Newton-Raphson method.

(b) Write a proram to solve an ordinary differential equation by modified Euler's method, $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 1$ at $y(0.2)$ and $y(0.4)$.

(c) Write a program on Lagrange's interpolation polynomial to find the value of a certain point from the given set of data. Find the value of 1.75 from the set of data :

x	1	1.5	2	3.2	4.5
y	5	8.2	9.2	11	16

Group - B

2. Answer any ***one*** of the following questions : 5×1=5

(a) Write a program to find the sum of the following series $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{N}$.

(b) Write a program to enter 100 integers into an array and sort them in an ascending order.

(c) Write a program to find the value of the integration by Trapezoidal rule, $\int_0^5 e^{-x}dx$ by taking 6 intervals.