বিদ্যাসাগর বিশ্ববিদ্যালয়

# VIDYASAGAR UNIVERSITY

## Question Paper

## B.Sc. General Examinations 2021

**(Under CBCS Pattern)**

## Semester - III

## Subject : MATHEMATICS

**Paper : SEC 1 - T**

**Full Marks : 40**

**Time : 2 Hours**

*Candidates are required to give their answers in their own words as far as practicable.*
*The figures in the margin indicate full marks.*



**[ THEORY OF EQUATIONS ]**

**(Theory)**

**Group-A**

Answer any ***three*** of the following questions : 12×3=36

1. (i) If $x^4 + px^2 + qx + r$ has a factor of the form $(x-\alpha)^2$, show that $8p^3 + 27q^2 = 0$ and $p^2 + 12r = 0$. 4

   (ii) Prove that the equation $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + .... ..... + \frac{x^n}{n!} = 0$ can not have a multiple root. 4

(iii) State Descartes' rule of sign. Apply it, find the number of positive and negative roots of the equation $x^4 + 4x^3 - x^2 - 10x + 3 = 0$. 4

2. (i) If the equation $ax^3 + 3bx^2 + 3cx + d = 0$ has two equal roots. Prove that $(bc - ad)^2 = 4(b^2 - ac)(c^2 - bd)$ and the equal root is $\dfrac{\frac{1}{2}(bc - ad)}{ac - b^2}$. 4

(ii) If the polynomial $x^n - qx^{n-m} + r$ has a factor of the form $(x - \alpha)^2$, show that $\left|\frac{q}{n}(n-m)\right|^n = \left|\frac{r}{n}(n-m)\right|^m$. 4

(iii) State and proof that an algebraic equation of degree has $n$ roots and no more. 4

3. (i) If $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots\ldots, \alpha_n$ be the roots of the equation $x^n + nax + b = 0$. Prove that $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)\ldots\ldots(\alpha_1 - \alpha_n) = n(\alpha_1^{n-1} + a)$. 4

(ii) If $\alpha$ be a multiple root of order 3 of the equation $x^4 + bx^2 + cx + d = 0 \, (d \neq 0)$, show that $\alpha = -\dfrac{8d}{3c}$. 4

(iii) Solve the equation $3x^4 + 20x^3 - 70x^2 - 60x + 27 = 0$, the roots being in Geometric Progression. 4

4. (i) If $\alpha, \beta, \gamma$ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of

(a) $\sum \alpha^2\beta^2$ (b) $\sum (\beta + \gamma - \alpha)^3$ 4

(ii) If the equation $x^4 + px^3 + qx^2 + rx + s = 0$ has roots of the form $\alpha \pm i\alpha, \beta \pm i\beta$ where $\alpha, \beta$ are real, prove that $p^2 - 2q = 0$ and $r^2 - 2qs = 0$ and hence solve the equation $x^4 + 4x^3 + 8x^2 - 24x + 36 = 0$. 8

5. (i) If $\alpha, \beta, \gamma$ be the roots of the equation $x^3 + qx + r = 0$, find the equation whose roots are $\frac{\beta}{\gamma}+\frac{\gamma}{\beta}, \frac{\gamma}{\alpha}+\frac{\alpha}{\gamma}, \frac{\alpha}{\beta}+\frac{\beta}{\alpha}$. 4

(ii) If $\alpha$ be a root of the equation $x^3 - 3x - 1 = 0$, prove that the other roots are $\alpha^2 - \alpha - 2,\ 2 - \alpha^2$. 4

(iii) Prove that the equation $(x+1)^4 = a(x^4+1)$ is a reciprocal equation if $a \neq 1$ and solve it when $a = -2$. 4

6. (i) Solve the equation $x^3 - 3x - 1 = 0$. 4

(ii) If $\alpha, \beta, \gamma$ be the roots of the equation $x^3 - 3qx + r = 0$, show that $(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha) = \pm\sqrt{27(4q^3 - r^2)}$. 4

(iii) Solve the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$. 4

**Group-B**

Answer any ***two*** of the following questions : 2×2=4

1. (i) Solve the equation $2x^3 - x^2 - 18x + 9 = 0$ if two of the roots are equal in magnitude but opposite in sign. 2

(ii) Expand $f(x) = x^4 - 4x^3 + 3x^2 + 3x + 7$ as a polynomial in $x - 1$. 2

(iii) State fundamental theorem of classical algebra. 2

(iv) Prove that the roots of the equation $\frac{1}{x-1}+\frac{2}{x-2}+\frac{3}{x-3} = x$ are all real. 2

***OR***

**[ LOGIC AND SETS ]**

**(Theory)**

**Group-A**

Answer any ***three*** of the following questions : 12×3=36

1. (i) If ρ be an equivalence relation on a set *S* and $a, b \in S$. Then prove that $cl(a) = cl(b)$ if and only if *a* ρ *b*. 4

   (ii) Find the equivalence classes determined by the equivalence relation ρ on ℤ defined by "*a* ρ *b* if and only *a* – *b* is divisible by 5" for $a, b \in \mathbb{Z}$. 4

   (iii) An equivalence relation ρ on a set *S* determines a partition of *S*. Conversely, each partition of *S* yields an equivalence relation on *S*. 4

2. (i) State and proof De Morgan's laws. 4

   (ii) Define power set. If *S* = {1, 2, 3}, then find the power set of *S*. 4

   (iii) Define cartesian product of a set. Then prove that

   $A \times (B \cap C) = (A \times B) \cap (A \times C)$ 4

3. (i) Examine whether the followings are is a tautology or not :

   $$\left(\left(A \Rightarrow (B \vee C)\right) \vee (A \Rightarrow B)\right)$$

   $$\left(\left(A \Leftrightarrow ((-B) \vee C)\right) \Rightarrow ((-A) \Rightarrow B)\right)$$ 8

   (ii) Prove that if *B* and $(B \Rightarrow C)$ are tautologies, then so on *C*. 4

4. (i) Defin a relation on a set. A relation ρ on the set *z* is given by $\rho = \{(a, b) \in z \times z \, : \, ab > 0\}$. Examine if it is equivalence relation or not. 1+4

   (ii) Let *A, B, C* are subsets of a universal set *S*. If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then prove that *B* = *C*. 3

(iii) For any three subsets $A$, $B$, $C$ of a universal set $S$, prove that $A \cap B = (A \cup B) \Delta (A \Delta B)$. 4

5. (i) What is biconditional statement? And define Associated Implications. 3+3

(ii) Draw the truth table and prove that :

(a) $\sim (p \wedge q) \equiv \sim p \vee \sim q$

(b) $(p \vee q) \Rightarrow r \equiv (p \Rightarrow r) \wedge (q \Rightarrow r)$ 3+3

6. (i) If $x^2$ is odd, then prove that $x$ must be odd. 2

(ii) $p : \forall$ real numbers $x$, $\cos x + \sin x = 1$

$\sim p : \exists$ a real number $x$ such that $\cos x + \sin x \neq 1$, prove that $\sim p$ is true. 4

(iii) Use method of contradiction prove that :

(a) If $n^3$ is odd, then $n$ is odd, $n$ being a positive integer.

(b) If $x$ and $y$ are integers such that $xy^2$ is even, then at least one of $x$, $y$ is even.

3+3

**Group-B**

Answer any ***two*** of the following questions : 2×2=4

1. (i) What are quantifiers in predicating logic?

(ii) Prove that the union of two reflexive relations on a set is a reflexive relation.

(iii) Write contrapositive statement for "If he has the courage, he will win."

(iv) Define conditional statement with truth table.

***OR***

**[ BOOLEAN ALGEBRA ]**

**(Theory)**

**Group-A**

Answer any ***three*** of the following questions : 12×3=36

1. (i) State the duality principle of Boolean algebra. 2

(ii) Show that the number of elements in a finite Boolean algebra is of power of 2. 3

(iii) Prove that there does not exist a Boolean algebra containing only three elements. 4

(iv) Prove that in a bounded distributive lattice an element can have at most one complement. 3

2. (i) Identify extreme elements in the following posets : 3+3

(a) The divisors of 60, ordered by divisibility.

(b) The set $\{a, b, c, d, e, f, g, h\}$, ordered like the subsets of $\{0, 1, 2\}$.

(ii) Let $(D_{12}, |)$ denote the poset of all divisors of 12. Show that $D_{12}$ is a lattice by drawing out the Hasse diagram for the poset and then verifying that each pair of divisors has both a meet and join. 6

3. (i) If $f$ is a function of three Boolean variables $x, y, z$ defined by $f(x,y,z) = xy + y'$. Express $f$ in disjunctive normal form. 5

(ii) Show that a lattice is distributive iff following identity holds : 4

$$(p \cap q) \cup (q \cap r) \cup (r \cap p) = (p \cup q) \cap (q \cup r) \cap (r \cup p)$$

(iii) Prove that a lattice is modular iff it satisfies $p \cup (q \cap (p \cup r)) = p \cup (r \cap (p \cup q))$ 3

4. (i) Define modular lattices and give an example of it. 2

(ii) In a Boolean algebra *B*, prove that for *a, b, c* in *B*. 3

$$(a+b+c).(a'+b+c).(a+b'+c).(a+b+c') = (b+c).(c+a).(a+b)$$

(iii) Describe the distinctions between Boolean algebra and the algebra of real numbers. 4

(iv) Prove that in a Boolean algebra, $a + a = a$ and $a \,.\, a = a$. 3

5. (i) Use NAND gate alone to represent the function $f(a,b,c,d) = (a \wedge b) \vee (c \wedge d)$. 3

(ii) Using Karnaugh Map, minimize the following boolean function

$$F(A,B,C,D) = \sum m(0,1,2,5,7,8,9,10,13,15).$$ 6

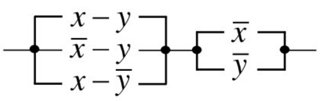
(iii) Draw the truth table for the Boolean function defined as

$$f(x_1, x_2, x_3) = x_1 \wedge (\sim x_2 \vee x_3).$$ 3

6. (i) Draw the circuit to realize the following Boolean function with simplified circuits

$$(x+\bar{y}+z)(x+yz)+\bar{z}w+w(\bar{y}+z).$$ 4

(ii) Find the function to represent the following circuit in simplified form




4

(iii) Show that the current will flow through the network by the Boolean function $[xy(\bar{x}y + x\bar{y})]$. 4

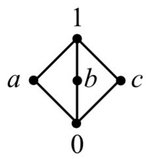
**Group-B**

Answer any ***two*** of the following questions : 2×2=4

1. (i) What are the basic digital logic gates?

(ii) Draw the $k$-map for $f(x, y) = xy + x\overline{y} + \overline{x}\,\overline{y}$.

(iii) What is the advantage of Quine McCluskey method?

(iv) Show that the lattice  is not distributive.



---